Dynamic Analysis of Transverse Crack in Rotor By Using Finite Element Method

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***Abstract*— Dynamic response analysis of a cracked rotor is attempted. The breathing of crack is accounted using the response dependent breathing crack model. Transient response of an accelerating cracked rotor is analyzed. Breathing behavior of the cracked rotor is analyzed and the effects of various parameters on the peak response variation and response of the cracked rotor is studied using time domain and frequency domain and plots.**

The influence of the presence of transverse cracks in a rotating shaft is analyzed. The crack has opening and closing on dynamic response during operation in the rotor. Initially a simple Jeffcott rotor is analyzed considering the lateral vibration. The dynamic response of the rotor with a breathing crack is evaluated by expanding the changing stiffness of the crack as a truncated using FEM. and then approach is based on the fact that the presence of a crack in rotating shaft reduces the stiffness of the structure, hence reducing the natural frequencies of the original uncracked shaft. This method is applied to compute various parametric studies including the effects of the crack depth and location on the dynamic of a crack rotor.

Using MATLAB commands 'bode' to get magnitude for given frequency range. Also, plot Frequency response function by using command 'loglog'.

***Keywords:* Cracked rotor, breathing crack, Transverse crack, Crack response.**

I. INTRODUCTION

An ever-increasing quest for higher power has lead to a highly stressed condition of rotating elements. Designs made to counter large centrifugal stresses in rotating members often lead to the problem of increased flexibility of rotors. Operating conditions also improve several thermal and mechanical stresses on these flexible rotors. An increased occurrence of fatigue cracks in these machines in recent times is an obvious outcome.

The occurrence of fatigue cracks is not only for machines that are at the end of the designed service life but also in new or overhauled rotors. Cracks in such cases are found because of faults in design or due to increased stress introduced by poor machining techniques, corrosion damage, several misalignment preloads and other factors. The worst part of the presence of cracks in such high power rotating machinery is the catastrophic nature of failure and the damage it brings about to the complete plant. The propagation rate of the crack is uncertain and sometimes the failure has taken place without enough warning. Propagating fatigue cracks can thus have deter mental effects on the reliability of rotating machinery. An early crack warning can considerably extend the durability of this very expensive machinery, increasing their reliability at the same time.

Vibration monitoring has a great potential in this regard since it can be carried out without dismantling any part of the machine and usually without taking the machine out of use. This is precisely the reason behind an increased attention the machine condition monitoring using vibration signature based techniques has been receiving in recent years. Possibility of losses running in to millions of dollars in case of downtime for even a day has made the reliability and availability of these high capacity machines a top most concern among engineers like never before.

Another trend in recent years regarding operating practices followed in the industry is to operate older machines, rather than replacing them with new ones, beyond their design life. This is done with life assessment and more aggressive monitoring techniques. This one again brings into focus the significance of the vibration based monitoring techniques. There are instance(Mayes and Davis,1980) where the saving in excess of £15,000,000 have been achieved due to extended operation of a cracked rotor with use of on load monitoring program(compared to scrapping the rotor for want of a replacement).

Recent larger power generating machines are more susceptible to various machine malfunctions including shaft cracks, according to an electric power research institute (EPRI) study. Apart from the public image and safety issue involving such catastrophic failures due to crack, it is the economics of situation that is more startling. According to another EPRI report, one utility paid $6.2 million purchase replacement power alone during an outage caused by a shaft crack on a turbine (Bently Nevada Technical Bulletin, 1986). The cost of repair and replacement power worked out to $800,000 per day in case. 50 failures have been reported during 1970 to 1985 alone. A couple of decades back one machine manufacturer has logged more than 28 incidents in North America over the period of 10 years in the power generation industry alone (Muszynska,1982). The manufacturer indicated that it was only a partial list. Obviously there could have been more such incidents in the past but they are never reported in open literature due to the adverse effect to the image of the manufacturers in the market.

Heavy damages in Germany had occurred in the nuclear power plant Wurgassen (Haas, 1977). For one year the plant had to be put out of operation because two new rotors had to be produced. Both the LP-rotors have had crack in the middle of the shaft. Like in Wurgassen an abnormal increase in the shaft vibration was noticed at many other cracked rotors (Schmerling and Hammon (1966), jack and Patterson (1977)). But in some cases the warning signs were too small and some rotor broke, in some cases turbine plants virtually exploded and fragments of the shafts flew away up to some hundred meters.

There is another example of crack appearing in the turbo generator. One of the 935 MW four pole units at Ontario Hydro's Darlington Nuclear Generating station was shut down due to the detection of a crack propagating through the generator rotor at the slip ring end (Sanderson, 1992). The existence of crack extending through 25% of the cross sectional area was detected shortly after reaching 50% of full power and with just 800 hours of accumulated operation.

Certainly one has to realize the fact that in practice cracks are comparatively rare. But if not detected in time these cracks may lead to catastrophic failure. As larger and more highly stressed rotating components are used in plant installed by electricity supply utilities throughout the world, the number of examples of turbo generator shaft cracking has increased. Moreover ,bulk of power equipment are approaching end of their design life and it is expected that failures due to fatigue initiated cracks will be a major causes of machine failures in the coming years.

II. LITERATURE REVIEW

A.s. Sekhar et al [1] has been studied on "crack identification in a rotor system: a model-based approach" the dynamics and diagnostics of cracked rotor have been gaining importance in recent years. In the present study a model-based method is proposed for the on-line identification of cracks in a rotor. The fault-induced change of the rotor system is taken into account by equivalent loads in the mathematical model. The equivalent loads are virtual forces and moments acting on the linear undamaged system to generate a dynamic behavior identical to the measured one of the damaged system. The rotor has been modeled using finite element method, while the crack is considered through local flexibility change. The crack has been identified for its depth and location on the shaft. The nature and symptoms of the fault, that is crack, are ascertained using the fast fourier transform.

Jean-jacques sinou et al [2] has been studied on "effects of a crack on the stability of a non-linear rotor system" the non-linear response and the various regions of instability of a non-linear cracked rotor system, for the first three critical speeds of the rotor. Moreover, in order to avoid the transition matrix computation associated with the floquet method, the stability analysis is carried out in the frequency domain, using a method in which a perturbation is applied to the known harmonic time-domain solution, which is calculated beforehand using the harmonic balance method. Firstly, a description is provided of the rotor system, and of the method used to model and incorporate the crack, together with a breathing mechanism. Secondly, the harmonic balance method, which is used to calculate the periodic response of the non-linear cracked rotor. The algorithm used to calculate the stability of these periodic solutions is then presented. Finally, the non-linear vibrational amplitudes of the cracked rotor are analyzed. Numerical examples are presented, in which the influence on the stability of the non-linear periodic response of a cracked rotor system are determined as a function of crack depth, crack location, support stiffness and disc position.

J. J. Sinou, a.w. Lees et al [3] has been studied on "the influence of cracks in rotating shafts" the influence of transverse cracks in a rotating shaft is analyzed. The two distinct issues of the changes in modal properties and the influence of crack breathing on dynamic response during operation. Moreover, the evolution of the orbit of a cracked rotor near half of the first resonance frequency is investigated. The results provide a possible basis for an on-line monitoring system. In order to conduct this study, the dynamic response of a rotor with a breathing crack is evaluated by using the alternate frequency/time domain approach. It is shown that this method evaluates the nonlinear behavior of the rotor system rapidly and efficiently by modeling the breathing crack with a truncated Fourier series. The dynamic response obtained by applying this method is compared with that evaluated through numerical integration. The resulting orbit during transient operation is presented and some distinguishing features of a cracked rotor are examined.

J.J. Sinou, A.W.Lees et al [4] has been studied on "A non-linear study of a cracked rotor" One form of damage that can lead to catastrophic failure is an undetected crack in a shaft and in recent years significant effort has been devoted to the detection of transverse cracks in shafts. This problem may be approached by observing the vibrational behavior of the rotor system with a crack and one of the advantages of vibration measurement is that the detection of crack can be carried out non-invasively. In this theoretical model is used to identify the characteristics of a system in the presence of a transverse crack. If a rotor has a crack, this crack may remain open or closed during the rotors motion, or it may open and close during different parts of the cycle. This latter case is referred to by some authors as a 'breathing crack', generally, two different approaches are attempted to identify the presence of a crack in rotating structures. The first approach is based on the fact that the presence of a crack in rotating shaft reduces the stiffness of the structure, hence reducing the natural frequencies of the original uncracked shaft. The second approach takes into account the influence of a transverse crack on the response of a rotor model.

J. S. RAO et al [5] has been studied "Theoretical analysis of lateral response due to torsional excitation of geared rotors" This is concerned with lateral response of geared rotors due to torsional excitation. Usually torsional response is determined due to excitation torque, such as short circuit torque, to deter-mine the safety of a turbo generator set and the coupling of bending and torsion in such an analysis is ignored. Such an analysis gives an accurate prediction only when the bending natural frequencies are not close to the spin speed and its higher as well as lower orders. In this work, the coupling between bending and torsion due to gears as well as the effect of axial torque on bending vibrations is taken into account to determine the coupled lateral and torsional response due to torsional excitation. Examples are given which illustrate the importance of such coupling mechanism giving rise to large amplitudes of bending vibrations which may be harmful to the operation of geared turbine rotors.

III. PROPOSED WORK

The system under study is illustrated in Fig. 4.1; the rotor is composed of a rotor shaft with one disc at the mid-span. The rotor shaft is modelled and discretized into 10 Timoshenko beam finite elements with four degrees of freedoms at each node and a constant circular cross-section. As illustrated in Fig. 4.1, the nodal displacement of a beam element is defined by $q = [v_1\ w_1\ \theta_1\ \psi_1\ v_2\ w_2\ \theta_2\ \psi_2]^T$
(1)

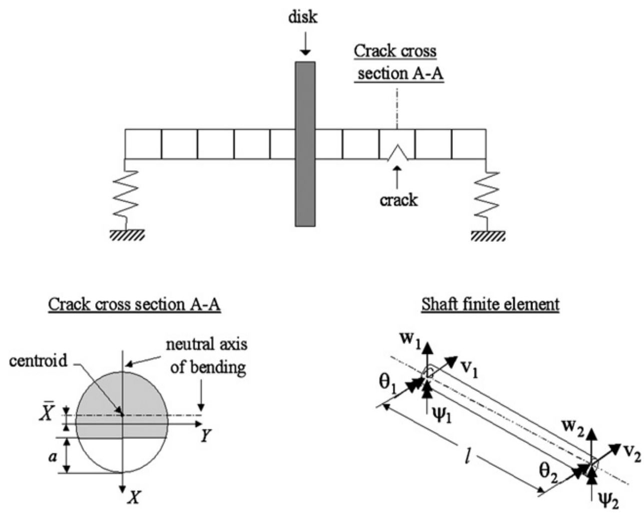

Fig.1. Rotor system and crack model cross-section.

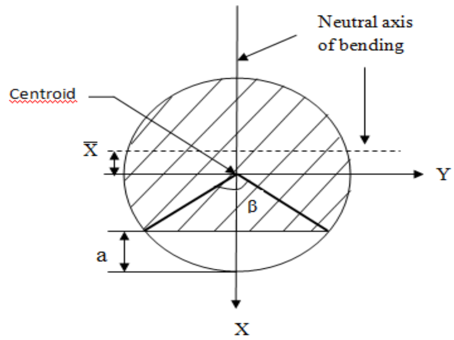

Fig.2. cross-section of crack element

A.MODELING OF BREATHING BEHAVIOUR OF CRACK

When the rotor is operating at a study state speed far way from critical speed and without any transient excitation, the breathing of crack can be approximated either by sinusoidal stiffness or stepwise stiffness fluctuation. However a truly breathing behavior can be taking into account the gradual opening and closing of the crack using stress intensity at the crack front at each instant and then finding the amount of crack opening and hence the stiffness. In this way, apart from getting a more accurate estimation of stiffness and more realistic representation of breathing, the model would be adaptable for all speed ranges and all type of excitation, steady as well as transient.

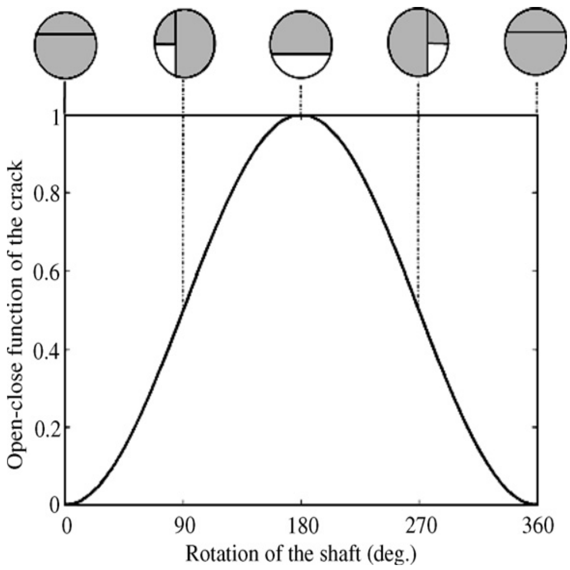

Fig.3.: Opening and closing of the crack

System equation of motion

The equation of motion of the complete rotor system in a fixedco-ordinate system can be written as

$$[M]\{\ddot{q}\}+[D]\{\dot{q}\}+[K]\{q\}=[F] \quad (2)$$

Where the mass matrix includes the rotary and translational mass matrices of the shaft, mass and diametral moments of the rigid disc. The matrix includes the gyroscopic moments and damping. The stiffness matrix considers the stiffness of the shaft elements and the bearing stiffness. Cracked element stiffness can be included easily, for the cracked rotor analysis. The excitation matrix consists of the unbalance forces due to disc having mass m_e, Eccentricity and the weight of the disc.

The discrete bearing stiffness coefficients are placed at the corresponding degrees of freedom and the equation of motion for the complete rotor system is defined as follows

$$[M_T+M_R]\{\ddot{q}\}+[C+\omega G]\{\dot{q}\}+[K]\{q\}=\{W+Q\} \quad (3)$$

Where C is damping matrix i.e C= α_d K.
α_d is a proportional factor to stiffness and defines the coefficient of damping.
G is the mass matrix including the shaft and rigid discs.
ω defines the rotational speed of the rotor.
Q and *W* define the vector of gravity force and unbalance force for the complete rotor system.

B. MODEL OF COMPLETE ROTOR

After assembling the different shaft elements, adding the modelling of the rigid discs and bearing, the complete rotor model with a crack is given by

$$[M]\{\ddot{q}\}+[C+\omega G]\{\dot{q}\}+[K-f(t)K_C]\{q\}=\{W+Q\} \quad (4)$$

Where *M*, *C*, *G* and *K* are the mass, damping, gyroscopic and stiffness matrices, respectively. *Q* and *W* define the vector of gravity force and imbalance force, respectively. K_C is the stiffness matrix due to the crack and *f (t)* the function representing the opening/closing effect.
The function describing the breathing crack may given by
$f(t) = 1/2(1 - \cos(\omega t))$ where ω is the rotational speed of the rotor. It may be observed that the crack is totally closed for
$f(t)=0$, and full open for $f(t) = 1$.

This chapter breathing crack model is studied. The rotor is composed of a rotor shaft with one disc at the mid-span. The rotor shaft is modelled and discretized into 10 Timoshenko beam finite elements with four degrees of freedoms at each node and a constant circular cross-section. And then finding the amount of crack opening and hence the stiffness. In this way apart from getting a more accurate estimation of stiffness and more realistic representation of breathing.

C. FORMULATING STIFFNESS AND MASS MATRICESFINITE ELEMENT MODELING OF BEAMS

The basic idea in the finite element is to find the solution of a complicated problem by replacing it by simpler one. The actual continuum or body is represented as an assemblage of subdivisions called finite elements. These elements are considered to be interconnected at specified joints called nodes or nodal points. The nodes usually lie on the element boundaries where adjacent elements are considered to be connected. Since the actual variation of the field variable like displacement, stress, temperature, pressure or velocity inside the continuum is not known, we assume that the variation of the field variable inside a finite element can be approximated by a shape function. These approximating functions (shape functions) are defined in terms of values of the field variable at the nodes. When field equations for whole continuum are written, the new unknowns will be the nodal values of the field variable. By solving the field equations, the nodal values of the field variable will be known. Once these are known, the approximating functions define the field variable throughout the assemblage of element. The finite element method is step-by-step implementation of following steps,

1) Discretization of structure.
2) Selection of a proper interpolation or displacement model.
3) Derivation of element stiffness, mass matrices and load vectors.
4) Assemblage of elemental equations to obtain overall equilibrium equations.
5) Solution for unknown displacements, frequencies or stresses and strains.

D. BEAM MODELING BY EULER BERNOULLI'S METHOD

The Euler – Bernoulli beam formulation is used to obtain the Eigen properties. Two degrees of freedom translation and slope are considered at each node of the beam element. Using the classical finite element approach, the element stiffness [K_e] and consistent mass matrices [M_e] for the beam element. The discrete bearing stiffness coefficients are placed at the corresponding degrees of freedom and the equation of motion for the complete rotor system is defined as follows

$$[M]\{\ddot{q}\} + [C + \omega G]\{\dot{q}\} + [K]\{q\} \quad (5)$$

where *M* and *G* are the mass and gyroscopic matrices including mass and gyroscopic matrices of the shaft and rigid discs. *C* and *K* are the external damping and stiffness matrices of the shaft. *Q* and *W* define the vector of gravity force and imbalance force for the complete rotor system.

$$[M]\{\ddot{q}(t)\} + [C + \omega G]\{\dot{q}(t)\} + [K]\{q(t)\} = \{F(t)\} \quad (6)$$

Modal reduction method

We know that final displacement equation is contribution of all mode. But contribution made by first few modes is much more than other. Therefore for our analysis we are considering only first few frequencies, for that purpose we are using modal reduction method. In this process we are considering all row of eigen vector [P] and no. of column equal to no. of modes we are considering. It helps to reduce computation time for MATLAB.

E. MODIFICATION IN MATRIX 'A' FOR SIMPLIFICATION PURPOSE BLOCK FORM

Apply row and column transformation to [A] such that we will get matrix [A] in following form, which will helpful for easiness in computation. Apply same row transformation on [B] and column transformation on [C] as applied on A.

Now , apply MATLAB command 'bode' to get magnitude for given frequency range. Also , plot Frequency response function by using command 'loglog', which gives FRF at any location on beam.

In this chapter have been studied on formulating stiffness and mass of cracked and uncracked rotor element and also studied on modal reduction method. state space formulation and finding global transfer function Now , apply MATLAB command 'bode' to get magnitude for given frequency range. Also, plot Frequency response function by using command 'loglog'.

IV. RESULTS AND DISCUSSIONS

The analysis has been carried out at steady state speed of 2200rpm. A crack at various locations and depth of crack varies shown in table .2, the non-linear vibration and their natural frequencies and attenuation of vibration amplitude is compared. The first six natural frequencies and their respective time response signal of beam configuration modeled using Euler-Bernoulli's and Timoshenko's beam theories. This numerical example presented, in which the influence on the non-linear response of cracked rotor system are determined as a function of crack depth, crack location.

Table .2 Values of the cracked pulsations (rad/s) of the rotor system versus the non-dimensional crack depth and the crack location L_{crack}

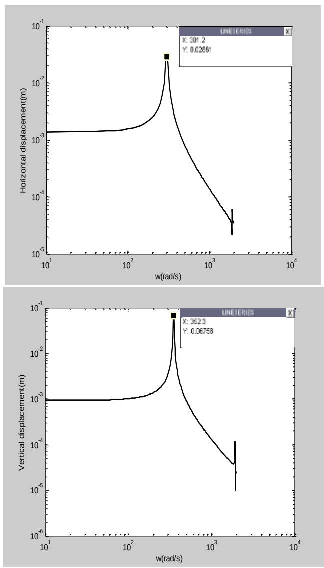

Fig.4. Horizontal and vertical steady-state responses of cracked rotor at 0.225m from left end (a=1)

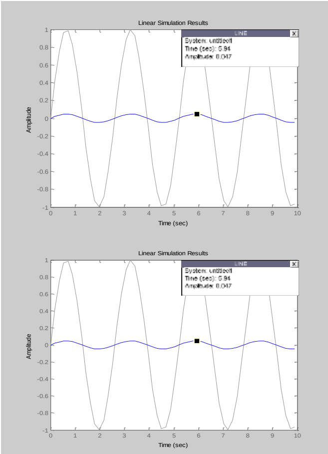

Fig.5. Horizontal and vertical time response signal of cracked rotor at 0.225m from left end (a=1)

The analysis has been carried out at steady state speed of 2200rpm. A crack at various locations and depth of crack varies shown in table .3 and table .4 it can be observed that the cracked pulsation is less than the uncracked pulsation.

Table.1. Values of the uncracked pulsations and the cracked pulsations (rad/s) of the rotor system versus the supports stiffness

S.NO	Stiffness (N/m)		ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
1	$2X10^5$		307	308	1478	1480	2251	2259
2	$5X10^5$		318	319	1747	1749	2885	2885
3	$2X10^6$		324	324	1913	1917	3405	3406
4	$2X10^7$		326	326	1965	1970	3573	3579
5		L_{cr} (m)	ω_1^{cr}	ω_2^{cr}	ω_3^{cr}	ω_4^{cr}	ω_5^{cr}	ω_6^{cr}
6	$2X10^5$	0.225	281	307	1469	1478	2144	2251
7	$5X10^5$	0.225	289	318	1729	1747	2727	2885
8	$2X10^6$	0.225	293	324	1887	1913	3183	3405
9	$2X10^7$	0.225	294	325	1937	1966	3329	3575

Table.2. Values of the uncracked pulsations and the cracked pulsations (rad/s) of the rotor system versus the disk position

S.NO	Disk Position (m)		ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
1	0.25		307	308	1478	1486	2251	2259
2	0.2		314	314	1403	1403	2342	2368
3	0.15		335	336	1259	1260	2478	2498
4	0.1		373	373	1131	1133	2366	2367
5		L_{cr} (m)	ω_1^{cr}	ω_2^{cr}	ω_3^{cr}	ω_4^{cr}	ω_5^{cr}	ω_6^{cr}
6	0.25	0.225	281	307	1469	1478	2144	2251
7	0.2	0.225	287	314	1394	1403	2227	2342
8	0.15	0.225	310	335	1256	1259	2375	2478
9	0.1	0.225	348	372	1126	1132	2249	2365

V. CONCLUSIONS

Vibration response analysis of cracked rotor has been carried out. Mostly a simple Jeffcott rotor model is used. However, a non-linear Finite Element based rotor dynamic code for the dynamic response analysis of cracked rotor is also developed. The code is particularly useful for studying coupling of vibration for a cracked rotor is also developed. Response dependent non-linear breathing crack model is used. Coupled vibration response has been studied with objective of obtaining crack specific features in the response. Finite element modeling result predictions by Euler-Bernoulli's beam theory is more accurate for lower modes while Timoshenko's theory with reduced order model predictions are more accurate at higher frequencies. Some of the analytical results are reported for a cracked rotor. Based on the work reported in the dissertation, following conclusions are

1. A non-linear breathing crack model helped in understanding the breathing behavior of the cracked rotor in finer detail during the model could give a better insight into the dynamics of a cracked rotor as well as the breathing behavior of crack.
2. The breathing behavior is shown depend upon unbalance, acceleration rate, crack depth and damping.
3. The dynamics of the cracked rotor during passage through critical speed and sub harmonic resonances. The breathing behavior of the rotor to be studied to gain better understanding of the dynamics of the rotor and obtain information for the purpose of crack detection. Nonlinear response dependent breathing crack model
4. Breathing behavior of the cracked rotor is analyzed and the effects of various parameters on the peak response variation and response of the cracked rotor is studied using time domain, frequency domain and plots.
5. Modifying stiffness matrix of a beam for representing a crack segment of the rotor in a FE model, this takes care of coupling mechanisms. To study this coupling in the case of rotating cracked shaft with the breathing cracked model.

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