STRESS ANALYSIS ON THICK WALLED CYLINDER WITHOUT AND WITH HOLES

G.Raju*, K. Hari Babu, V.Vana Kiran Kumar
Department of Mechanical Engineering, Narasaraopet Engineering College, Narasaraopet, Andhra Pradesh, India-522601 *Corresponding Email: raju.guduri@gmail.com

***Abstract*—The stress is directly proportional to strain up to yield point Beyond elastic point, particularly in thick walled cylinders. The operating pressures are reduced or the material properties are strengthened. There is no such existing theory for the stress distributions around radial holes under impact of varying internal pressure. Present work puts thrust on this area and relation between pressure and stress distribution is plotted graphically based on observations. Here focus is on pure mechanical analysis & hence thermal, effects are not considered. The thick walled cylinders with a radial cross-hole ANSYS Macro program employed to evaluate the fatigue life of vessel. Stresses that remain in material even after removing applied loads are known as residual stresses.**

Elasto plastic analysis with bilinear kinematic hardening material is performed to know the effect of hole sizes. It is observed that there are several factors which influence stress intensity factors. The Finite element analysis is conducted using commercial solvers ANSYS & CATIA. Theoretical formulae based results are obtained from MATLAB programs. The results are presented in form of graphs and tables.

Key Words: Pressure, Stress, Strain, Analysis.

1. INTRODUCTION

Thick walled cylinders are widely used in chemical, petroleum, military industries as well as in nuclear power plants .They are usually subjected to high pressures & temperatures which may be constant or cycling. Industrial problems often witness ductile fracture of materials due to some discontinuity in geometry or material characteristics. The conventional elastic analysis of thick walled cylinders to final radial & hoop stresses is applicable for the internal pressures up to yield strength of material. But the industrial cylinders often undergo pressure about yield strength of material. Hence a precise elastic-plastic analysis accounting all the properties of material is needed in order to make a full use of load carrying capacity of the material & ensure safety w.r.t strength of cylinders.

1.1. Finding residual stresses:

Stresses that remain in material even after removing applied loads are known as residual stresses. These stresses occur only when material begins to yield plastically. Residual stresses can be present in any mechanical structure because of many causes. Residual stresses may be due to the technological process used to make the component. Manufacturing processes lead to plastic deformation.
In our case as the material enters Elastic-Plastic state, upon removing the loads, there exists a difference of stresses measured during loading and unloading times. Even theoretical formulas are available; it needs to verify the maximum stresses induced using FEM. Today there exists a vast scope to sue the FEM for analysis of the same.

1.2. Finding relations between various parameters in analysis of cylinders with holes:

With respect to the literature review, work has been not done to find fundamental equations depicting relationship between various parameters (pressure vs. stress) for thick-walled cylinders with radial holes. Here attempt has been made to find a graphical relationship of the same based on results and observations obtained.

1.3. OBJECTIVES OF THE WORK

The following are the principal objectives of the work

1. Stress analysis of thick walled cylinders with radial holes & understand the effect of relative dimensions/parameters of hole on equivalent stress developed due to internal pressure.

2. Study of Autofrettage process & find out the residual stresses theoretically & using FEM Method by considering bilinear kinematic hardening state (elasto-plastic state), for uniform cylinder as well as cylinder with radial hole.

3. Depicting relationship between internal pressure applied and equivalent stress graphically for

Proceedings of International Conference on Recent Trends in Mechanical Engineering-2K15(NECICRTME-2K15), 20th – 21st November,2015

elastic-plastic cases of uniform cylinder as well as cylinder with radial holes.

2. LITERATURE REVIEW

Xu& Yu [1] Carried down shakedown analysis of an internally pressurized thick walled cylinders, with material strength differences. Through elasto-plastic analysis, the solutions for loading stresses, residual stresses , elastic limit , plastic limit & shakedown limit of cylinder are derived.

Hojjati&Hossaini[2] studied the optimum auto frottage pressure & optimum radius of the elastic-plastic boundry of strain-hardening cylinders in plane strain & plane strain conditions . They used both theoretical &Finite element (FE) modeling. Equivalent von-Mises stress is used as yield criterion.

Ayub et al.[3] presented use of ABAQUS FE code to predict effects of residual stresses on load carrying capacity of thick walled cylinders.

Zheng&Xuan [4] carried out autofrettage& shake down analysis of power-law strain hardening cylinders S.T thermo mechanical loads. Closed form of FE solutions & FE modeling were employed to obtain optimum autofrettage pressure under plain strain & open-ended conditions.

Lavit& Tung [5] solved the thermoelastic plastic fracture mechanics problem of thick walled cylinder subjected to internal pressure and non-uniform temperature field using FEM. The correctness of solution is provided by using Barenblatt crack model.

In the present work, Elasto plastic analysis of thick cylinder with bilinear kinematic hardening material is performed to know the effect of hole sizes.

3. PRESSURE LIMITS OF THICK WALLED CYLINDERS

In general thick walled cylinders are subjected to internal pressure, as shown in Fig 1, which cause radial and hoop stress distributions across the thickness (Assuming geometric linearity in material).

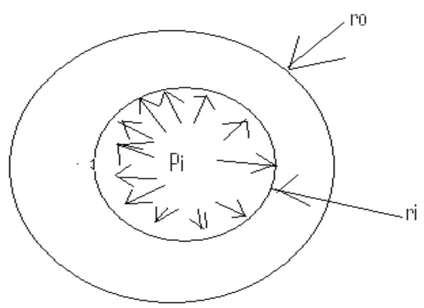

Fig 1. Cylinder under internal pressure

There exist a set of equations which give us relationship between Internal pressure and stresses developed, derived from above mentioned equations (1), (2),(3), which are in turn derived from lame's equations of thick cylinder.

Consider a plain strain cylinder internal radius & outer radius r_i & outer radius r_o. When pressure Pi is large enough the cylinder begins to yield from surface $r = r_i$. There exists a radius rc at the elastic and elastoplastic boundary interface. The associated pressure is Pc. So the material can be analysed as region between $r_i < r < r_c$ and r_c < r < r_o. The first one is in plastic state and second being in elastic state.

3.1. ELASTIC STATE:

$\sigma_a = (p_i r_i^2 - p_o r_o^2)/(r_o^2 - r_i^2)$ (4)

where

σa= stress in axial direction (MPa, psi)

pi= internal pressure in the tube or cylinder (MPa, psi)

po= external pressure in the tube or cylinder (MPa, psi)

r_i = internal radius of tube or cylinder (mm, in)

r_o = external radius of tube or cylinder (mm, in)

The stress in circumferential direction at a point in the tube or cylinder wall can be expressed as:

σ_θ = [(pi r_i^2 - por_o^2) / (r_o^2 - r_i^2)] - [r_i^2 r_o^2(po- pi) / r^2(r_o^2 - r_i^2)]

(5)

where σ_θ = stress in circumferential direction (MPa, psi)

The stress in tangential direction at a point in the tube or cylinder wall can be expressed as:

σ_r = [(pir_i^2- por_o^2) / (r_o^2 – r_i^2)] + [r_i^2 r_i^2(po- pi) / r^2(r_o^2 – r_i^2)]

(6)

where σ_r = Radial stress in tangential direction.

The strain components are as follows

$$\varepsilon_r = \left(\frac{1+v}{E}\right)\left(\frac{p_c r_c^2}{r_o^2} - r_c^2\right)\left(1 - \frac{r_o^2}{r^2} - 2v\right) \quad (7)$$

$$\varepsilon_\theta = \left(\frac{1+v}{E}\right)\left(\frac{p_c r_c^2}{r_o^2 - r_c^2}\right)\left(1 - 2v + \frac{r_o^2}{r^2}\right) \quad (8)$$

3.2. ELASTIC-PLASTIC STATE:

The governing equations in formulating stress for elastic-plastic region have been derived by considering power-law hardening model, strain gradient(modified von-mises)theory[14] for axi-symmetric problem .

$$\sigma_\theta - \sigma_r = \frac{r\partial\sigma_r}{\partial r} \quad (9)$$

$$r\left(\frac{\partial\varepsilon_\theta}{\partial r}\right) = \varepsilon_r - \varepsilon_\theta \quad (10)$$

From above equations, employing classical plasticity solution, final useful equations we get is:

$$p_i=\left(\frac{\sigma_y}{\sqrt{3}}\right)\left[\left(1-\frac{r_c^2}{r_o^2}\right)\ 2\ln\frac{r_c}{r_i}\right] \quad (11)$$

$$\sigma_r=\left(\frac{\sigma_y}{\sqrt{3}}\right)\left[-1+\frac{r_c^2}{r_o^2}-2\ln\frac{r_c}{r_i}\right] \quad (12)$$

$$\sigma_\theta=\left(\frac{\sigma_y}{\sqrt{3}}\right)\left[1+\frac{r_c^2}{r_o^2}-2ln\frac{r_c}{r}\right] \quad (13)$$

Where σ_y the yield strength of material. And is p_i is the internal pressure applied. Here main assumption in that external applied pressure/load is zero.

3.3. ANALYSIS OF AUTOFRETTAGE PROCESS

Residual stresses induced (both tension as well as compression) in thick cylinders due to internal pressure application forcing the maximum equivalent stress to cross the yield point. This is autofrettage phenomenon. The fatigue.

The pressure to initiate auto frottage is known as autofrettage pressure. Pa

$$p_A=\frac{\sigma_y}{2}\left[1-\frac{m^2}{k^2}\right]+\sigma_y\ \ln m \quad (14)$$

3.3.1 Stress distribution under autofrettage pressure loading

$$\sigma_r=\sigma_y\left[\ln\left(\frac{r}{R_p}\right)-\left(\frac{1}{2}\right)\left(1-\frac{R_p^2}{r_o^2}\right)\right] \quad (15)$$

$$\sigma_r=\sigma_y\left[\ln\left(\frac{r}{R_p}\right)-\left(\frac{1}{2}\right)\left(1-\frac{R_p^2}{r_o^2}\right)\right] \quad (16)$$

Above equations give radial and hoop stresses for an autofrettage phenomenon.

3.3.2. Residual stress distributions

It is assumed that during unloading the material follows HOOKE's law & the pressure is considered to be reduced (applied in negative pressure) elastically across the whole cylinder. Residual stress after unloading can then be obtained by removing Autofrettage pressure load elastically across the whole cylinder.

The dotted lines show the unloading distribution curves and solid lines show the loading distribution curves

$$\sigma_r=p_a\left[\frac{1-\frac{r_o^2}{r^2}}{k^2-1}\right] \quad (17)$$

$$\sigma_\theta=p_a\left[\frac{1-\frac{r_o^2}{r^2}}{k^2-1}\right] \quad (18)$$

Where k $={}^{r_o}/_{r_i}$, m $={}^{r_p}/_{r_i}$, $R_p=\sqrt{(r_i*r_o)}$, Pa is the autofrettage pressure.

The elastic stresses developed during loading condition can be given as

$$\sigma_\theta=\sigma_y\left[1+ln\left(\frac{r}{R_p}\right)-\left(\frac{1}{2}\right)\left(-\left(\frac{R_p^2}{r_o}\right)\right)\right]$$

$$for\ r_i\le r\le R_p \quad (19)$$

$$\sigma_\theta=\frac{\sigma_y R_p^2}{2r_o^2(r_o^2-R_p^2)}\left[1-\left(\frac{r_o^2}{r^2}\right)\right]$$

$$for\ R_p\le r\le r_o \quad (20)$$

$$\sigma_r=\sigma_y\left[ln\frac{r}{R_p}\left(\frac{1}{2}\right)\left(1-\frac{R_p^2}{r_o^2}\right)\right]$$

For $r_i\le r\le R_p$ (21)

$$\sigma_r=\frac{\sigma_y R_p^2}{2r_o^2\left(r_o^2-R_p^2\right)}\left[1+\frac{r_o^2}{r^2}\right]$$

For $R_p\le r\le r_o$ (22)

$$\sigma_{res\ hoop}=\sigma_{\theta\ unloading}-\sigma_{\theta\ loading} \quad (23)$$

$$\sigma_{res\ radial}=\sigma_{r\ unloading}-\sigma_{r\ loading} \quad (24)$$

No yielding occurs due to residual stresses. Superimposing these distributions on the previous loading distributions allow the two curves to be subtracted both for the hoop and radial stress and produce residual stresses.

3.3.3. CYLINDERS WITH RADIAL HOLES

The elastic hoop stress concentration factor is defined as the ratio of maximum principal stress & lame's hoop stress on the inside surface of the pressurized cylinder

SCF $=\frac{\sigma_{max}}{\sigma_{lame}}$. (25)

For the cylinder with wall ratio k=1/β & internal pressure p, the reference stress is 15

$$\sigma_{lame}=\left(\frac{k^2+1}{k^2-1}\right)$$

SCF is a measure of relative influence of cross hole& may be used to define the peak loads for cyclic loading.

SCF= Actual stresses (with holes) / theoretical stresses (without holes).

4. FINITE ELEMENT MODEL

In most cases of uniform cylinders theoretical stress relations are available that is uniform cylinders operated within elastic and plastic pressure regions

K=$\iint B^T.D.B\ dr.d\theta$.

B= strain displacement matrix.

D = stress strain matrix
K= stiffness matrix.

4.1 THE GEOMETRY AND MATERIAL PROPERTIES CONSIDERED

The dimensions of the steel cylinder taken:
R_i= 330 mm and R_o= 500 mm

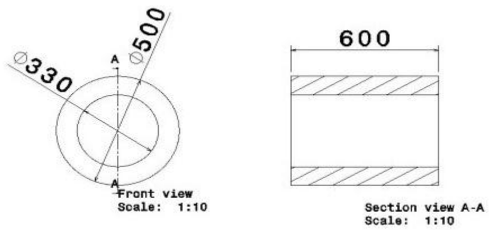

Fig 2: Thick wall Cylinder

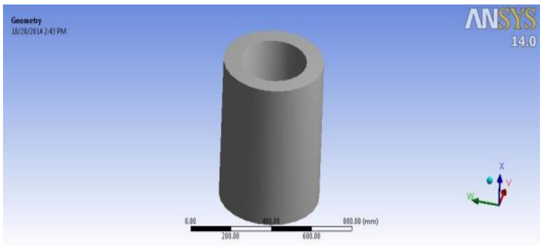

Fig 3. CATIA model of thick wall cylinder without holes

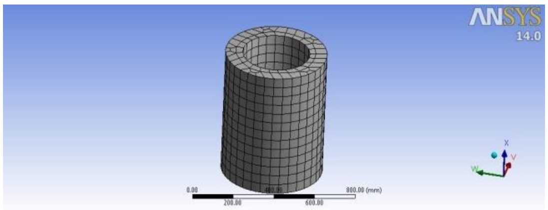

Fig 4. Meshed view of thick wall cylinder without holes

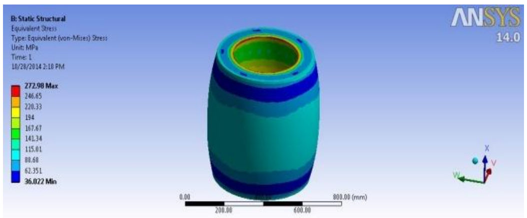

Fig 5. Stress distribution of thick wall cylinder without holes

Length can be of any dimension, as it is a case of axe-symmetric plain strain problem. We have chosen 600 mm.
Geometrically the entire cylinder is uniform (across the cross section also), material is isotropic in nature.
Entire analysis work has been done assuming /neglecting thermal effects.
For the cylinder with holes case, the hole is a radial cross bore of dimension R_h=40 mm is chosen.
The following material properties are chosen.

YOUNG'S MODULUS: 200 GPA
Poisson ratio: 0.3
Yield strength: 684 MPA.

The main criteria for failure chosen are maximum strain energy criterion or **von misses failure criteria.** It says that the material will fail when the equivalent stress exceeds the yield point limit. The main criteria for failure chosen are maximum distortion energy criterion or **von Misses yield criteria.**
It says that the material will fail when the equivalent stress exceeds the yield point limit.
For an axe-symmetric problem there are no shear forces. Hence hoop, longitudinal and radial stresses are the principal stresses.

$$(1/2)((\sigma_\theta - \sigma_r)^2 - (\sigma_r - \sigma_z)^2 - (\sigma_z - \sigma_\theta)^2) \leq \sigma_y^2 \quad (1)$$

4.2. ELASTIC ANALYSIS OF THICK WALLED CYLINDERS.

4.2.1. ANALYSIS OF UNIFORM CYLINDERS

Cylinder is then subjected to an internal pressure varying gradually (increased in steps) and corresponding maximum von Misses stress values are noted from the analysis results. The iterative procedure is continued till the von Misses stress reaches near about yield strength values
Specified dimensions are chosen and modelled in the software CATIA. The assumptions are made:

1. Cylinder without end-caps, subjected to internal pressure.
2. Material is perfectly elastic.
3. Default tetrahedral mesh gives enough accuracy.

Theoretical stresses based on lame's equations for elastic analysis are used to validate CATIA outputs.
The general lame's equations are followed for elastic analysis by theory which is shown in mathematical modelling chapter.

$$\sigma_{eq} = \sqrt{[\sigma_\theta^2 + \sigma_r^2 - \sigma_r . \sigma_\theta]}$$

(2)

This is internal pressure required at the onset of yielding of inner bore surface. That is the load to initiate the plasticity at the internal cylinder radius, often expressed as Elastic load capacity

$(\gamma_0 = \frac{p_o}{\sigma_y})$

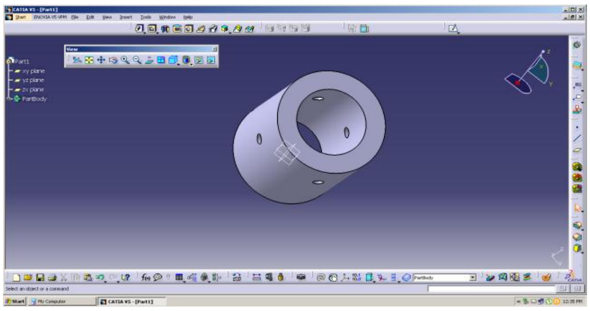

Fig 6. CATIA model of thick wall cylinder with four holes

Load capacity of a cylinder:
Where
γ_0 is the load capacity;
β is the radius ratio;(R_i/R_o)
Po is the pressure where plasticity begins at internal walls of cylinder.
σ_y is the yield strength of material.

$$\gamma_0 = \frac{1-\beta^2}{\sqrt{3}} = \frac{p_0}{\sigma_y}$$

Where is the load capacity; is the radius ratio $\frac{R_i}{R_0}$;
Po is the pressure where plasticity begins at internal walls of cylinder and σ_y is the yield strength of material.
For the above specified dimensions,
β = 0.66,
$\sigma_y = 684$ M pa
Hence
p_o=$[1-0.66^2]/\sqrt{3} * 684\ Mpa220.8Mpa$.

The internal pressure at the inner surface is applied from a starting value of 75 Mpa & slowly is incremented in steps of 5 Mpa . In each case the corresponding maximum equivalent stress is tabulated as depicted in Table-2. A screenshot at one of pressures in CATIA is shown in Fig 7.

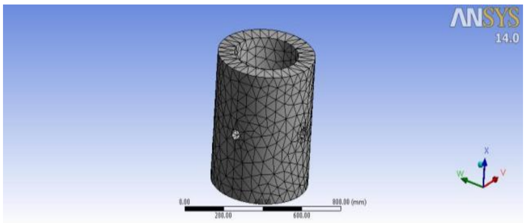

Fig 7. Meshed view of thick wall cylinder with four holes

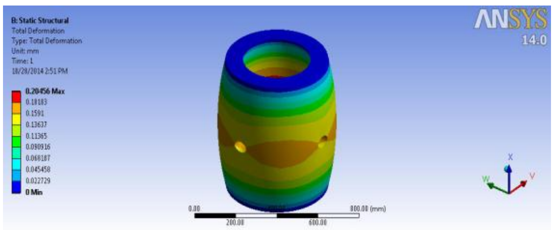

Fig 8. Stress distribution of thick wall cylinder with four holes

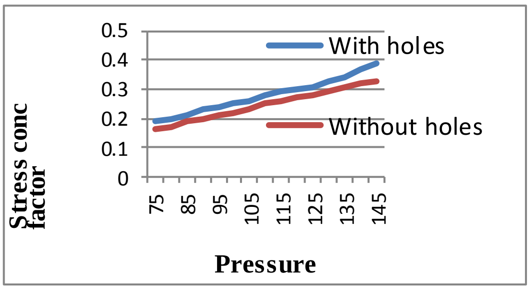

Fig 9. Stress concentration factor

Table-1: Variation of Internal Pressure

Pressure (M pa)	SCF without holes	SCF with holes$\left(\sigma_{\theta h}/\sigma_{\theta wh}\right)$
75	0.17911	0.19178
80	0.19105	0.20456
85	0.20299	0.21735
90	0.21493	0.23013
95	0.22687	0.24292
100	0.23880	0.25570

4.2.2. ELASTIC ANALYSIS OF THICK WALLED CYLINDER WITH A RADIAL HOLE

As this is again the elastic analysis, expected relationship between pressure and stress should be the same. Now only slope of graph will change as the pressures required to attain maximum stresses are lower. The Fig 8 shows the screenshot of CATIA model with radial hole considered. The internal pressure is varied & corresponding equivalent stresses are measured. Fig 7 shows the stress variation with pressure for with & without holes within elastic limits.

4.3.ELASTIC-PLASTIC ANALYSIS.

When cylinder is loaded to such pressures, yielding begins at inner wall. So here the relative pressures load that initiates the plastic state from inner wall is obtained from earlier elastic analysis. Using theoretical relations, the hoop & radial stress distributions during loading & unloading are generated according to a simple MatLab program (Table.2). The outputs of the program are shown in Fig 10.

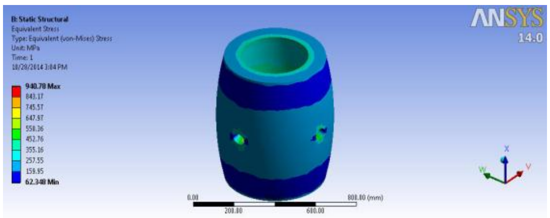

Fig 10. Stress distribution of thick wall cylinder with four holes

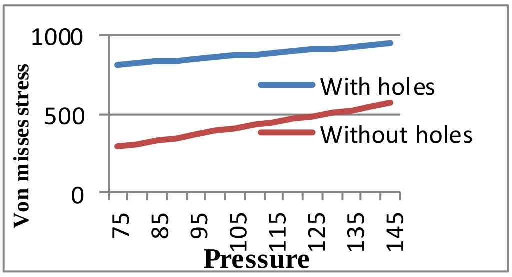

Fig 11. Stress Distribution of 4 holes

Table-2: Variation of Stress

Pressure (M pa)	Max Von misses stress (M pa) without hole	Max Von misses stress(M pa) with hole
75	292.48	814.87
80	311.98	825.30
85	331.48	835.15
90	350.98	846.71
95	370.48	856.04
100	389.98	864.70

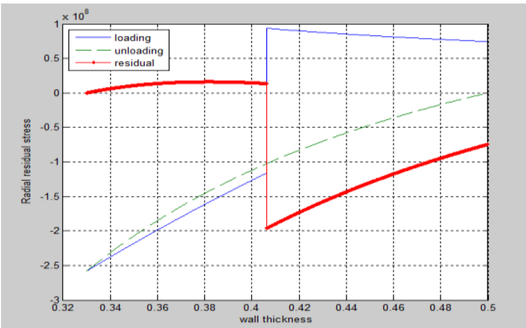

Fig 12. Residual Stress without hole

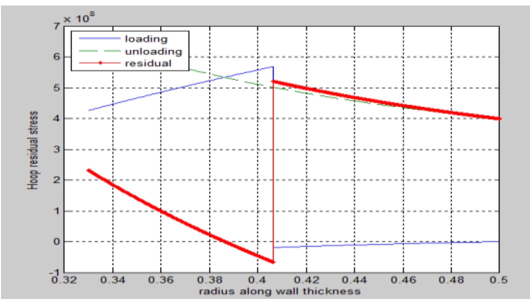

Fig 13. Residual Stress with hole

4.4.ELASTOPLASTIC ANALYSIS OF CYLINDER WITH RADIAL HOLE.

The analysis is carried out in Finite Element Method using ANSYS. A cylindrical segment is loaded by internal pressure on the internal surface and along the radial hole. An 8 noded solid -45 three dimensional element is employed to mesh the segment. The three surfaces were applied with symmetry boundary conditions an axial thrust

$Q = p * \frac{\beta^2}{1-\beta^2}$ is applied at the 4th surface, simulates reactions of cylinder heads. Fig .5 shows the meshed model of the segment in Ansys. Pressure is varied slightly & corresponding stress distribution along the hole surface. It is observed that unlike uniform cylinder the higher stresses are noticed at the same pressure values.

5. CONCLUSIONS

5.1 SUMMERY

An attempt has been made to know the load capacity of a cylinder with radial holes. The work is organized under elastic & elastic-plastic analysis. Classical book work formulas have been employed to obtain the stress distribution in cylinder without holes subjected to internal pressure. Being a new problem the elastoplastic analysis of cylinders with radial hole, there were no theoretical relations. Based on available finite element models, three dimensional analyses have been carried out to predict the actual stress behaviour along the cylinder wall especially at the cylinder bore. MATLAB, CATIA & ANSYS software have been used as per requirements.

5.2. FUTURE SCOPE OF WORK

The results can be compared with standard codes available. The unloading behaviour of thick walled cylindrical pressure vessel with holes is another extension for this work.

The load cycles can be increased to know local plastic shakedown limit.ANSYS command level code may be developed to carry out this shakedown analysis also. Finally the effect of hole dimensions as well as cylinder wall thickness on the maximum stresses induced may be modelled using nural network.

References

[1] S.Xu&M.Yu, "Shakedown Analysis of thick walled cylinders subjected to internal pressure with the inified strength criterion", International Journal of Pressure vessels & piping, Vol 82, pp706-712 ,2005.

[2] M.H.Hojjati&A.Hossaini, "Theoretical& Finite Element Modelling of autofrettage process in strain hardening thick walled cylinders", International Journal of Pressure vessels & piping, vol 84, pp 310-319, 2007

[3] A.B .Ayub, M.N. Tanim& M.K. Elbasheer, "pressure limits of Thick walled cylinders", proc.

International Multiconference of Engineers & Computer Scientists, Vol 2, Mar 18-20, 2009, Hongkong.

[4] X.T Zheng& F.Z Xuan, "Autofrettage& Shakedown analysis of strain hardening cylinders under thermo-mechanical loadings", Journal of strain analysis, Vol-46, pp 45-55, 2011.

[5] I.M Lavit& N.U. Trung, "Thermoelastoplastic deformations of a thick walled cylinder with a radial crack", Journal of Applied Mechanics & Technological Physics, Vol 49, pp 491-499, 2008.