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Spectral Decimation Functions and Forbidden Eigen Values in the Graph of Level Sierpinski Triangles

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Abstract

The physical phenomena occurring in the objects represented by nonlinear models are analyzed on fractal domains. Laplacian-type operator for functions on fractal domain plays an significant role in studying the nonlinear partial differential equations on fractals. The concept of graph energy described on the finite connected graph is specified. A work about the graph energy is mainly concerned on a Level Sierpinski Triangles. The harmonic functions of the Level Sierpinski Triangles are constructed by iterated function system. The Spectral decimation function for the Level Sierpinski Triangles are determined using the standard Laplacian. The Laplacian Renormalization constant and the forbidden eigenvalues on the Level Sierpinski Triangles are examined.

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Keywords: Iterated function System, Energy renormalization constant, Spectral decimation function, Laplacian renormalization factor, forbidden eigenvalue.

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1. Introduction

The term "fractal" was first used by mathematician Benoit Mandelbrot in 1975. The formal mathematical definition of fractal is defined by Benoit Mandelbrot says that a fractal is a set for which the Hausdorff Besicovich dimension strictly exceeds the topological dimension [1]. However, this is a very abstract definition. Generally, we can define a fractal as a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Fractals are generally self-similar and independent of scale[2]. The Sierpinski discovered a set called the Sierpinski gasket(SG) at the beginning of this century[3].

The Laplacian was first defined on the Sierpinski gasket SG as the generator of a diffusion process[4]. Later the Laplacian theory extended to nested fractals by Lindstrom[7] and to post critically finite (p.c.f.) fractals i.e. a finite connected self similar fractals by Kigami [6]. The eigenvalues of the Laplacian can be obtained through an iterative process called spectral decimation method, and which was introduced by R.Strichartz [5] and developed by

Shima [8]. The eigenvalues of the Laplacian on the Sierpinski fractal were analysed by M.Fukushima[9]. In this paper, first we determine the spectral decimation function on the sierpinski triangle. The energy renormalization constant on the graph of Sierpinski triangle is calculated and the Laplacian renormalization factor is calculated. Through the normalized Laplacian the all forbidden eigen values on the sierpinski triangle are determined..

2. Mathematical definitions

2.1 Spectral Decimation function

The Laplacian operator on a post critically finite (p.c.f.) fractal accepts spectral decimation, if there exists a rational function R, a finite set A and a constant λ>1 such that all eigen values of Δ can be written in the form

$$\lambda^m \lim_{n\to\infty} \lambda^n R^{-n}(w),\ w \in A,\ m \in N \tag{1}$$

2.2. Laplacian on Finite Graphs

For any set S, we use $l(S)$ to denote the set of real valued functions on S and

$$l_0(V_m) = \{f \in l(V_m) : f(p) = 0 \ \ for \ \ p \in V_0\} \tag{2}$$

For two sets U and V, we define $L(U,V) = \{A: l(U) \to l(V) \ \ and \ \ A \ is \ linear\}$

In particular, L(U) means L(U,U).

2.3 Vertex degree

Let G be a simple graph with the set of vertices V(G) and the set of edges E(G). We say that two vertices x, y are neighbours if they are connected by exactly one edge, denoted by e(x, y) in the graph.

2.4 Standard and Normalized Laplacian

Given a function $f \in l(V(G))$, the graph (standard) Laplacian of f at a vertex x is defined as

$$\Delta f(x) = \sum_{e(x,y)\in V(G)} (f(x) - f(y)) \tag{3}$$

And the normalized (probability) Laplacian is defined by

$$\hat{\Delta} f(x) = \frac{1}{\deg(x)} \sum_{e(x,y)\in V(G)} (f(x) - f(y)) \tag{4}$$

The symmetric matrix D corresponding to Δ is called the Laplacian matrix and it has the expression

$$D_{i,j} = \begin{cases} 1 & if \ i \neq j \\ -\deg(x_i) & if \ i = j \ \ and\ e(x_i, x_j) \in E(G) \\ 0 & if \ \ otherwise \end{cases} \tag{5}$$

$for \ \ x_i, x_j \in V(G).$

The normalized Laplacian on Vm satisfies

$$\hat{\Delta}_m f(x) = \frac{H_m f(x)}{\hat{\mu}_m(x)} \quad for \ \ f \in l(V_m) \tag{6}$$

Where $H_m = \sum_{w \in W_m} R_w^t \, D \, R_w \quad and \quad \hat{\mu}_m(x) = \sum_{w \in w_m} R_w^t (-D) R_w$

$also \ \hat{\mu}_m(x) = \deg(x) \, and = (-H_m).$

The Laplacian matrix D is considered to be a symmetric matrix with row sums zero, and have positive entries in the off diagonal and negative entries in the on diagonal. We choose the measure factor r such that $r = (r_1^{-1}, r_2^{-1}, r_3^{-1}, \dots, r_s^{-1}) \in l(S) \quad and \quad r_0^{-1} = \sum_{s \in S} r_s^{-1}.$

(H_m,r) is the generalized standard Laplacian with weight r on the graph G_m. We decompose the matix H_m into

$$H_m = \begin{pmatrix} T_m & J_m^t \\ J_m & X_m \end{pmatrix} \qquad (7)$$

Where

$T_m \in L(V_0), \ J_m \in L(V_0, V_m^0) \ and \ X_m \in L(V_m^0).$ In particular $T=T_1$, $J=J_1$ and $X=X_1$.

3. Constructions of the Level Sierpinski Triangles

3.1 The Level-2 Sierpinski Triangle

In an equilateral triangle, each side is bisected and four triangles are formed. Three triangles are retained and the one triangle is deleted. Repeating this process again and again the level-2 sierpinski triangle is formed as the limit set.

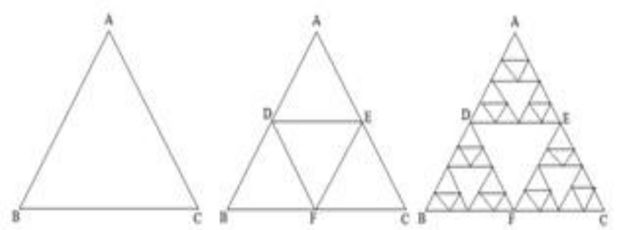

Figure1: Level-2 Sierpinski Gasket-G_0,G_1 and SG_2

3.2. The Level-3 Sierpinski Triangle

In an equilateral triangle, each side is trisected and nine triangles are formed. Six triangles are retained and the three triangles are deleted. Repeating this process again and again the level-3 sierpinski triangle is formed as the

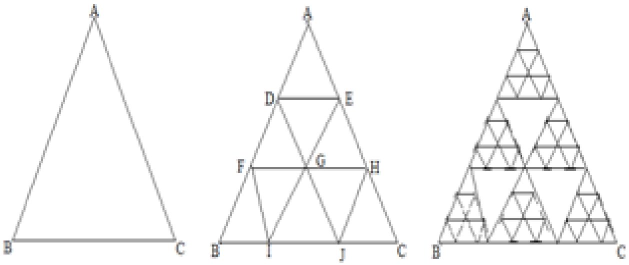

Figure 2: Level-3 Sierpinski Gasket-G_0,G_1 and SG_3

limit set .

The level-3 sierpinski triangle(SG3) is obtained by the iteration process and is made up of six triangles, each triangle is similar to the whole with contraction ratio 1/3.

3.3 The Level-4 Sierpinski Triangle

In an equilateral triangle, each side is divided into four equal segments and sixteen triangles are formed. Ten triangles are retained and the six triangles are deleted. Repeating this process again and again the level-4 sierpinski triangle is formed as the limit set.

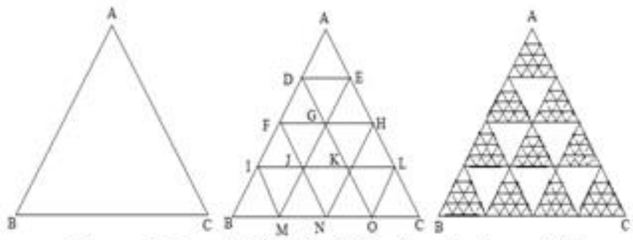

Figure 3: Level-4 Sierpinski Gasket-G_0,G_1 and SG_4

4. Energy Renormalization Constant

The generalized Laplacian is said to have a strong harmonic structure if there exist rational function $K_D(\lambda)$ and $K_T(\lambda)$ such that $X+\lambda M$ is invertible, then

$$T - J^T (X+\lambda M)^{-1} J = K_D(\lambda) D + K_T(\lambda) T \qquad (8)$$

Where $K_D(0)$ is the energy renormalization constant, and the diagonal matrices is defined as $M_{i,i} = - X_{i,i.}$

Let D be the Laplacian matrix on G_0. Then

$$D=\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

4.1. The Level-2 Sierpinski Triangle

Using the graphs G_0 and G_1 of the level-2 sierpinski gasket (Figure:1) and equation(5), the matrices are determined as follows:

The measure factor is $r = (1,1,1)$

$$H = \begin{pmatrix} -2 & 0 & 0 & 0 & 1 & 1 \\ 0 & -2 & 0 & 1 & 0 & 1 \\ 0 & 0 & -2 & 1 & 1 & 0 \\ 0 & 1 & 1 & -4 & 1 & 1 \\ 1 & 0 & 1 & 1 & -4 & 1 \\ 1 & 1 & 0 & 1 & 1 & -4 \end{pmatrix}$$

Hence $T = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ $J = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

$$X = \begin{pmatrix} -4 & 1 & 1 \\ 1 & -4 & 1 \\ 1 & 1 & -4 \end{pmatrix}$$

$$T - J^T (X + \lambda M)^{-1} J = \frac{1}{(4\lambda - 5)(2\lambda - 1)} \begin{pmatrix} -2(8\lambda^2 - 12\lambda + 3) & 3 - 2\lambda & 3 - 2\lambda \\ 3 - 2\lambda & -2(8\lambda^2 - 12\lambda + 3) & 3 - 2\lambda \\ 3 - 2\lambda & 3 - 2\lambda & -2(8\lambda^2 - 12\lambda + 3) \end{pmatrix}$$

$$K_D(\lambda) = \frac{3 - 2\lambda}{(4\lambda - 5)(2\lambda - 1)} \; and \; K_T(\lambda) = \frac{2\lambda}{2\lambda - 1}$$

Hence the energy renormalization factor is $K_D(0)=3/5$.

And the renormalized graph energy ε_m at level m is defined by

$$\varepsilon_m(f) = \alpha^{-m} E_m(f)$$

For ε_m and E_m are bilinear,

$$\varepsilon_m(f, g) = \alpha^{-m} E_m(f, g)$$

where α is called the renormalization factor.

Hence the renormalized graph energy is $\varepsilon_m(f) = \left(\frac{3}{5}\right)^{-m} E_m(f)$.

4.2 The Level-3 Sierpinski Triangle

Using the graphs G_0 and G_1 of the level-3 sierpinski gasket (Figure:2) and equation(5), the matrices are determined as follows:

The measure factor is $r = (1,1,1,1,1,1)$

The matrix corresponding to the standard Laplacian on G_1 is

$$H = \begin{pmatrix} -2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & -4 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & -4 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -6 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & -4 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & -4 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & -4 \end{pmatrix}$$

Hence from the equation (7),

$$T = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix} \; J^T = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix},$$

And then

$$T - J^T (X + \lambda M)^{-1} J$$

$$= \frac{1}{6(128\lambda^5 - 640\lambda^4 + 1208\lambda^3 - 1048\lambda^2 + 396\lambda - 45)} \begin{pmatrix} -1536\lambda^5 + 7296\lambda^4 - 1286\lambda^3 + 10120\lambda^2 - 3252\lambda + 252 & 48\lambda^3 - 200\lambda^2 + 276\lambda - 126 & 48\lambda^3 - 200\lambda^2 + 276\lambda - 126 \\ 48\lambda^3 - 200\lambda^2 + 276\lambda - 126 & -1536\lambda^5 + 7296\lambda^4 - 1286\lambda^3 + 10120\lambda^2 - 3252\lambda + 252 & 48\lambda^3 - 200\lambda^2 + 276\lambda - 126 \\ 48\lambda^3 - 200\lambda^2 + 276\lambda - 126 & 48\lambda^3 - 200\lambda^2 + 276\lambda - 126 & -1536\lambda^5 + 7296\lambda^4 - 1286\lambda^3 + 10120\lambda^2 - 3252\lambda + 252 \end{pmatrix}$$

And from equation (8),

$T - J^T (X+\lambda M)^{-1} J = K_D(\lambda) D + K_T(\lambda) T$, we get,

$$K_D(\lambda)=\frac{48\lambda^3-200\lambda^2+276\lambda-126}{6(128\lambda^5-640\lambda^4+1208\lambda^3-1048\lambda^2+396\lambda-45)}$$

And

$$K_T(\lambda)=\frac{1536\lambda^5-7296\lambda^4+12768\lambda^3-9720\lambda^2+2700\lambda}{12(128\lambda^5-640\lambda^4+1208\lambda^3-1048\lambda^2+396\lambda-45)}$$ Hence the energy renormalization factor is $K_D(0)=7/15$.

Hence the renormalized graph energy is $\varepsilon_m(f)=\left(7/15\right)^{-m}E_m(f)$.

4.3 The Level-4 Sierpinski Triangle

Using the graphs G_0 and G_1 of the level-4 sierpinski gasket (Figure:3) and equation(5), the matrices are determined as follows:

The measure factor is r = (1,1,1,1,1,1,1,1,1,1)

The matrix corresponding to the standard Laplacian on G_1 is

$$H_1=\begin{pmatrix}
-2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1\\
1 & 0 & 0 & -4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 1 & -4 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & -4 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 1 & 1 & -6 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 & 1 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & -6 & 1 & 0 & 1 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & -6 & 1 & 0 & 1 & 1\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 0 & 0 & 1\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -4 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & -4 & 1\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & -4
\end{pmatrix}$$

The Value of is $T-J^T(X+\lambda M)^{-1}J$ calculated.

And from equation (8),

$T - J^T (X+\lambda M)^{-1} J = K_D(\lambda)\, D + K_T(\lambda)\, T$, we get,

$$K_D(\lambda) = \frac{\begin{array}{l} -576\lambda^5 + 3712\lambda^4 - 63521\lambda^3 \\ +11992\lambda^2 - 7506\lambda + 1845 \end{array}}{\begin{array}{l} 36864\lambda^8 - 288768\lambda^7 + 959488\lambda^6 \\ -1753536\lambda^5 + 1907344\lambda^4 \\ -12429488\lambda^3 + 459784\lambda^2 \\ -82863\lambda + 4635 \end{array}}$$

And

$$K_T(\lambda) = \frac{\begin{array}{l} 36864\lambda^8 - 27955\lambda^7 + 894208\lambda^6 \\ -1560704\lambda^5 + 1600880\lambda^4 - 907563\lambda^3 \\ +311622\lambda^2 - 41700\lambda \end{array}}{\begin{array}{l} 36864\lambda^8 - 288768\lambda^7 + 959488\lambda^6 \\ -1753536\lambda^5 + 1907344\lambda^4 - 12429488\lambda^3 \\ +459784\lambda^2 - 82863\lambda + 4635 \end{array}}$$

Hence the energy renormalization factor is $K_D(0)=(41/103)=0.39805$.

Hence the renormalized graph energy is $\varepsilon_m(f) = \left(41/103\right)^{-m} E_m(f)$.

5. Laplacian Renormalization Factor

The spectral decimation function is defined by

$$R(\lambda) = \frac{\lambda - K_T(\lambda)}{K_D(\lambda)}$$

5.1 The Level-2 Sierpinski Triangle

$$R(\lambda) = \lambda(5 - 4\lambda)$$

Definition By Shima[8], The spectral decimation function R(λ) satisfies R(0)=0 and $R'(0) = \frac{r_0^{-1}}{K_D(0)}$. Where r_0^{-1} is a measure factor. Here R(0)=0 and and L.H.S. $R'(0)$ =5 and R.H.S. $\frac{r_0^{-1}}{K_D(0)} = \frac{3}{3/5} = 5$

Hence the Laplacian renormalization constant is 5.

5.2 The Level-3 Sierpinski Triangle

$$R(\lambda)=\frac{384\lambda^6-2304\lambda^5+5448\lambda^4-6336\lambda^3+3618\lambda^2-810\lambda}{24\lambda^3-100\lambda^2+138\lambda-63}$$ Here R(0)=0 and and L.H.S. $R'(0)$ =12.857 and

R.H.S. $\frac{r_0^{-1}}{K_D(0)}=\frac{6}{7/15}=12.857$

Hence the Laplacian renormalization constant is 12.857.

5.3 The Level-4 Sierpinski Triangle GLE

$$R(\lambda)=\frac{36864\lambda^9-325632\lambda^8+1239040\lambda^7-2647744\lambda^6+3468048\lambda^5-2843828\lambda^4+1367347\lambda^3-394485\lambda^2+46335\lambda}{-576\lambda^5+3712\lambda^4-63521\lambda^3+11992\lambda^2-7506\lambda+1845}$$ H

Here R(0)=0 and and L.H.S. $R'(0)$ =25.114 and

R.H.S. $\frac{r_0^{-1}}{K_D(0)}=\frac{10}{41/103}=25.119$

Hence the Laplacian renormalization constant is 25.11.

6. Normalized Laplacian

6.1. The Level-2 Sierpinski Triangle

The normalized Laplacian $\hat{\Delta}_0$ *and* $\hat{\Delta}_1$ for G_0 and G_1 of the level-2 sierpinski gasket are defined as

$\hat{\Delta}_0=-T^{-1}D \textit{ and } \hat{\Delta}_1=-W^{-1}H$,

where the diagonal matrix W= $\begin{pmatrix}-T & 0\\ 0 & M\end{pmatrix}$

Hence we have

$$\hat{\Delta}_0=\begin{pmatrix}-1 & 1/2 & 1/2\\ 1/2 & -1 & 1/2\\ 1/2 & 1/2 & -1\end{pmatrix} W=\begin{pmatrix}2&0&0&0&0&0\\0&2&0&0&0&0\\0&0&2&0&0&0\\0&0&0&4&0&0\\0&0&0&0&4&0\\0&0&0&0&0&4\end{pmatrix}$$

$$\hat{\Delta}_1 = \begin{pmatrix} 1 & 0 & 0 & -1/2 & -1/2 & 0 \\ 0 & 1 & 0 & -1/2 & 0 & -1/2 \\ 0 & 0 & 1 & 0 & -1/2 & -1/2 \\ -1/4 & -1/4 & 0 & 1 & -1/4 & -1/4 \\ -1/4 & 0 & -1/4 & -1/4 & 1 & -1/4 \\ 0 & -1/4 & -1/4 & -1/4 & -1/4 & 1 \end{pmatrix}$$

6.2 The Level-3 Sierpinski Triangle

The normalized Laplacian $\hat{\Delta}_0$ *and* $\hat{\Delta}_1$ for G_0 and G_1 of the level-3 sierpinski gasket are

Hence we have $\hat{\Delta}_0 = \begin{pmatrix} -1 & 1/2 & 1/2 \\ 1/2 & -1 & 1/2 \\ 1/2 & 1/2 & -1 \end{pmatrix}$

$$\hat{\Delta}_1 = \begin{pmatrix} 1 & 0 & 0 & -1/2 & -1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1/2 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1/2 & 0 & -1/2 \\ -1/4 & 0 & 0 & 1 & -1/4 & -1/4 & -1/4 & 0 & 0 & 0 \\ -1/4 & 0 & 0 & -1/4 & 1 & 0 & -1/4 & -1/4 & 0 & 0 \\ 0 & -1/4 & 0 & -1/4 & 0 & 1 & -1/4 & 0 & -1/4 & 0 \\ 0 & 0 & 0 & -1/6 & -1/6 & -1/6 & 1 & -1/6 & -1/6 & -1/6 \\ 0 & 0 & -1/4 & 0 & -1/4 & 0 & -1/4 & 1 & 0 & -1/4 \\ 0 & -1/4 & 0 & 0 & 0 & -1/4 & -1/4 & 0 & 1 & -1/4 \\ 0 & 0 & -1/4 & 0 & 0 & 0 & -1/4 & -1/4 & -1/4 & 1 \end{pmatrix}$$

6.3 The Level-4 Sierpinski Triangle

The normalized Laplacian $\hat{\Delta}_0$ *and* $\hat{\Delta}_1$ for G_0 and G_1 of the level-4 sierpinski gasket are

$$\hat{\Delta}_0 = \begin{pmatrix} -1 & 1/2 & 1/2 \\ 1/2 & -1 & 1/2 \\ 1/2 & 1/2 & -1 \end{pmatrix}$$

$$\hat{\Delta}_1 = \begin{pmatrix} 1 & 0 & 0 & -1/2 & -1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1/2 & 0 & 0 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/2 & 0 & 0 & -1/2 \\ -1/4 & 0 & 0 & 1 & -1/4 & -1/4 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/4 & 0 & 0 & -1/4 & 1 & 0 & -1/4 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/4 & 0 & 1 & -1/4 & 0 & -1/4 & -1/4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3/18 & -3/18 & -3/18 & 1 & -3/18 & 0 & -3/18 & -3/18 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1/4 & 0 & -1/4 & 1 & 0 & 0 & -1/4 & -1/4 & 0 & 0 & 0 \\ 0 & -1/4 & 0 & 0 & 0 & -1/4 & 0 & 0 & 1 & -1/4 & 0 & 0 & -1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3/18 & -3/18 & 0 & -3/18 & 1 & -3/18 & 0 & -3/18 & -3/18 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3/18 & -3/18 & 0 & -3/18 & 1 & -3/18 & 0 & -3/18 & -3/18 \\ 0 & 0 & -1/4 & 0 & 0 & 0 & 0 & -1/4 & 0 & 0 & -1/4 & 1 & 0 & 0 & -1/4 \\ 0 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & -1/4 & 0 & 0 & 1 & -1/4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & -1/4 & 0 & -1/4 & 1 & -1/4 \\ 0 & 0 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & -1/4 & 0 & -1/4 & 1 \end{pmatrix}$$

7. Forbidden Eigen Values

7.1 The Level-2 Sierpinski Triangle

We denote $\psi = \{\lambda \in R : K_D(\lambda) = 0 \, or \, \det(X + \lambda M) = 0\}$

And the elements in ψ are the forbidden eigen values.

$\psi_k = \{\lambda \in \psi : \lambda \, is \, an \, eigen \, value \, of \, -\hat{\Delta}_k\}$

And the elements in ψ_k are the initial eigen values at step k or forbidden eigen values at step k.

The forbidden eigen values are obtained by considering $K_D(\lambda) = 0$ or det(X+λM)=0 This gives the values of λ, that is 1.5. Hence the value of ψ is 1.5. To find the forbidden eigen values at the initial step or step 1 is an eigen value of $\hat{\Delta}_1$ and also in ψ.

The eigen values of -$\hat{\Delta}_1$ are 0,0.75,0.75,1.5,1.5,1.5

The forbidden eigen value at the initial step is 1.5.

7.2 The Level-3 Sierpinski Triangle

Det(X+λM)=0 gives the values of λ, they are 1.5,1.5,1.1667. Hence the values of ψ are 1.5,1.5,1.1667.

The eigen values of $\hat{\Delta}_1$ are 0,1.5,1,0.3964,0.3964,1.1036,1.1036,1.5,1.5,1.5

Hence the forbidden eigen values at the initial step are 1.5 and 1.5.

7.3 The Level-4 Sierpinski Triangle

Det(X+λM)=0 gives the values of λ, they are 1.5, 1.5, 1.322, 1.3085, 1.3059, 1.25, 1, 0.9071, 0.9071, 0.4524, 0.4524, 0.0945.

The eigen values of $\hat{\Delta}_1$ are 0,0.6667, 0.2390, 0.2390, 0.7753, 0.7753, 1.1523, 1.1523, 1.25, 1.25, 1.5, 1.5, 1.5, 1.5, 1.5.

The forbidden eigen values at the initial step are 1.5, 1.5 and 1.25.

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